ADAPTIVE CONTROL AND SYNCHRONIZATION OF BAROTROPIC MODEL

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Abstract

In this paper, study adaptive control and synchronization of barotropic model. First, study the stability of equilibrium point of barotropic model. Then control the chaotic behavior of barotropic model to its equilibrium point using adaptive control method. Finally, study chaos synchronization of barotropic model using adaptive control method.

1. Introduction

Chaos in control systems and controlling chaos in dynamical systems have both attracted increasing attention in recent years. A chaotic system has complex dynamical behaviors that posses some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space. Controlling chaos has focused on the nonlinear systems such as a barotropic model. Non-divergent barotropic model is

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$$\frac{\partial \nabla^2 \psi}{\partial t} = -J(\psi, \nabla^2 \psi) - \beta \frac{\partial \psi}{\partial x}, \tag{1}$$

where ψ is the streamfunction, $J(\psi, \nabla^2 \psi)$ is the Jacobian, β is the beta parameter.

2. Adaptive Control Chaos of Barotropic Model

Feedback control method is applied to achieve this goal. Consider the controlled system of (1), which has the form

$$\frac{\partial \nabla^2 \psi}{\partial t} = -J(\psi, \nabla^2 \psi) - \beta \frac{\partial \psi}{\partial x} + u, \tag{2}$$

where u is external control input, which will drag the chaotic trajectory $\nabla^2 \psi$ of the barotropic model to equilibrium point $E = \overline{\nabla^2 \psi}$, which is the steady states E_0.

In this case, the control law is

$$u = -g(\nabla^2 \psi - \overline{\nabla^2 \psi}),$$

where g (estimate of g^*, respectively) is updated according to the following adaptive algorithm.

$$\dot{g} = \mu(\nabla^2 \psi - \overline{\nabla^2 \psi})^2,$$

where μ is adaption gains. Then, the controlled system (2) has following form

$$\frac{\partial \nabla^2 \psi}{\partial t} = -J(\psi, \nabla^2 \psi) - \beta \frac{\partial \psi}{\partial x} - g(\nabla^2 \psi - \overline{\nabla^2 \psi}). \tag{3}$$

Theorem 2.1. *For $g < g^*$, the equilibrium point $E = \overline{\nabla^2 \psi}$ of the system* (3) *is asymptotically stable.*

Proof. Let us consider the Lyapunov function

$$V(\xi) = \frac{1}{2}\left[(\nabla^2 \psi - \overline{\nabla^2 \psi})^2 + \frac{1}{\mu}(g - g^*)^2\right].$$

The time derivative of V in the neighbourhood $E = \overline{\nabla^2\psi}$ of the system (3) is

$$\dot{V} = (\nabla^2\psi - \overline{\nabla^2\psi})\nabla^2\dot{\psi} + \frac{1}{\mu}(g - g^*)\dot{g}.$$ (4)

By substituting (3) in (4),

$$\dot{V} = (\nabla^2\psi - \overline{\nabla^2\psi})\left[-J(\psi, \nabla^2\psi) - \beta\frac{\partial\psi}{\partial x} - g(\nabla^2\psi - \overline{\nabla^2\psi})\right] + (g - g^*)(\nabla^2\psi - \overline{\nabla^2\psi})^2.$$

Let $\eta = (\nabla^2\psi - \overline{\nabla^2\psi})$, since $\overline{\nabla^2\psi}$ is an equilibrium point of the uncontrolled system (1), $\dot{V}$ becomes

$$\dot{V} = -\eta J(\psi, \nabla^2\psi) - \eta\beta\frac{\partial\psi}{\partial x} - g\eta^2 + (g - g^*)\eta^2.$$

It is clear that for positive parameters J, β, μ, if we choose $g < g^*$, then $\dot{V}$ is negative semidefinite. Since, V is positive definite and $\dot{V}$ is negative semidefinite, $\eta, g \in L_\infty$. From $\dot{V}(t) \le 0$, we can easily show that the square of η are integrable with respect to t, namely, $\eta \in L_2$. From (3), for any initial conditions, we have $\dot{\eta} \in L_\infty$. By the well-known Barbalat's lemma, we conclude that $\eta \to 0$ as $t \to +\infty$. Therefore, the equilibrium point $E = \overline{\nabla^2\psi}$ of the system (3) is asymptotically stable.

3. Adaptive Synchronization of the Barotropic Model

Consider two nonlinear systems

$$\dot{x} = f(t, x),$$ (5)

$$\dot{y} = g(t, y) + u(t, x, y),$$ (6)

where $x, y \in R^n$, $f, g \in C^r[R^+ \times R^n, R^n]$, $u \in C^r[R^+ \times R^n \times R^n, R^n]$, $r \ge 1$, R^+ is the set of non-negative real numbers. Assume that (5) is the drive system, (6) is the response system, and $u(t, x, y)$ is the control vector.

In this section, we consider adaptive synchronization barotropic model. This approach can synchronize the chaotic systems, when the parameters of the drive system are fully unknown and different with those of the response system. Assume that there are two barotropic model, such that the drive system is to control the response system. The drive and response system are given, respectively, by

$$\frac{\partial \nabla^2 \psi_1}{\partial t} = -J(\psi_1, \nabla^2 \psi_1) - \beta \frac{\partial \psi_1}{\partial x},$$
(7)

where the parameter J, β are unknown or uncertain, and

$$\frac{\partial \nabla^2 \psi_2}{\partial t} = -J_1(\psi_2, \nabla^2 \psi_2) - \beta_1 \frac{\partial \psi_2}{\partial x} - u_1,$$
(8)

where J_1, β_1 are parameters of the response system, which need to be estimated, and $u = [u_1]^T$ is the controller, we introduced in (8). We choose

$$u_1 = k_1 e_{\nabla^2 \psi},$$
(9)

where $e_{\nabla^2 \psi}$ are the error states, which are defined as follows

$$e_{\nabla^2 \psi} = \nabla^2 \psi_2 - \nabla^2 \psi_1.$$
(10)

Theorem. *Let u_1 be the control and k_1 can be chosen so that the following inequalities holds,*

$$P = k_1 > 0.$$
(11)

Then the two barotropic models (7) *and* (8) *can be synchronized under the adaptive controls* (9).

Proof. It is easy to see from (7) and (8), that the error system is

$$\dot{e}_{\nabla^2 \psi} = -J_1(\psi_2, \nabla^2 \psi_2) - \beta_1 \frac{\partial \psi_2}{\partial x} + J(\psi_1, \nabla^2 \psi_1) + \beta \frac{\partial \psi_1}{\partial x} - u_1.$$
(12)

Let $e_J = J_1 - J$, $e_\beta = \beta_1 - \beta$. Choose the Lyapunov function as follows

$$V(t) = \frac{1}{2} e_{\nabla^2 \psi}^2.$$

Then, the differentiation of V along trajectories of (12) is

$$\dot{V}(t) = e_{\nabla^2\psi}\dot{e}_{\nabla^2\psi}$$

$$= e_{\nabla^2\psi}\left[-J_1(\psi_2, \nabla^2\psi_2) - \beta_1\frac{\partial\psi_2}{\partial x} + J(\psi_1, \nabla^2\psi_1) + \beta\frac{\partial\psi_1}{\partial x} - u_1\right]$$

$$= -e_{\nabla^2\psi}\left[J_1(\psi_2, \nabla^2\psi_2) + \beta_1\frac{\partial\psi_2}{\partial x} - J(\psi_1, \nabla^2\psi_1) - \beta\frac{\partial\psi_1}{\partial x} + u_1\right]$$

$$= -e_{\nabla^2\psi}\left[J_1(\psi_2, \nabla^2\psi_2) - J(\psi_1, \nabla^2\psi_1) + J(\psi_2, \nabla^2\psi_2) - J(\psi_2, \nabla^2\psi_2)\right]$$

$$\quad - e_{\nabla^2\psi}\left[\beta_1\frac{\partial\psi_2}{\partial x} - \beta\frac{\partial\psi_1}{\partial x} + \beta\frac{\partial\psi_2}{\partial x} - \beta\frac{\partial\psi_2}{\partial x}\right] - e_{\nabla^2\psi}u_1$$

$$= -e_{\nabla^2\psi}e_J + e_{\nabla^2\psi}J(\psi_1, \nabla^2\psi_1) - e_{\nabla^2\psi}J(\psi_2, \nabla^2\psi_2)$$

$$\quad - e_{\nabla^2\psi}e_\beta\frac{\partial\psi_2}{\partial x} + e_{\nabla^2\psi}\beta\frac{\partial\psi_1}{\partial x} - e_{\nabla^2\psi}\beta\frac{\partial\psi_2}{\partial x} - k_1 e^2_{\nabla^2\psi}$$

$$\leq -e_J e_{\nabla^2\psi} - J(\psi_2, \nabla^2\psi_2)e_{\nabla^2\psi} - e_\beta\frac{\partial\psi_2}{\partial x}e_{\nabla^2\psi} - \beta\frac{\partial\psi_2}{\partial x}e_{\nabla^2\psi} - k_1 e^2_{\nabla^2\psi}$$

$$\leq -k_1 e^2_{\nabla^2\psi}$$

$$= -e^T Pe,$$

where $e = [|e_{\nabla^2\psi}|]^T$ and P is as in (11). Since, $V(t)$ is positive definite and $\dot{V}(t)$ is negative semidefinite, it follows that $e_{\nabla^2\psi}$, J, $\beta \in L_\infty$. From $\dot{V}(t) \leq -e^T Pe$, we can easily show that the square of $e_{\nabla^2\psi}$ are integrable with respect to t, namely, $e_{\nabla^2\psi} \in L_2$. From (12), for any initial conditions, we have $\dot{e}_{\nabla^2\psi} \in L_\infty$. By the well-known Barbalat's lemma, we conclude that $e_{\nabla^2\psi} \to 0$ as $t \to +\infty$. Therefore, in the closed-loop system, $\nabla^2\psi_2(t) \to \nabla^2\psi_1(t)$ as $t \to +\infty$. This implies that the two barotropic model have synchronized under the adaptive controls (9).

4. Conclusions

In this paper, we first give sufficient conditions of parameters that make equilibrium point of the barotropic model (1) using adaptive control to be asymptotically stable. Finally, we give sufficient conditions of parameters that make equilibrium point of synchronization of the barotropic model using adaptive control to be asymptotically stable.

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